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The Value of Air Force Sabbaticals to Operations, Academia, and Cadets

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The Value of Air Force Sabbaticals to Operations, Academia, and Cadets

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Introduction

From 2002 through 2011, five civilian faculty members at the USAFA Department of Mathematical Sciences (DFMS), including the four authors of this paper, did a one-year sabbatical at Headquarters, Air Force Space Command (HQ AFSPC) Analysis Division (A9).

Moore [4] summarizes the types of problems that DFMS faculty members addressed at HQ AFSPC/A9: "Challenges include solving problems in the areas of predictive analysis for current and future space systems, constellation optimization for increased military utility, operational effects of terrain masking, jamming and innovative mitigation tactics, techniques and procedures (TT & Ps), and assessments of the military utility of improved space weather observation and forecast systems." HQ AFSPC/A9 has sponsored these sabbaticals and appreciated the expertise that DFMS personnel have brought to their operational mission. The second section describes each of the author's work and the benefits HQ AFSPC/A9 has realized from their efforts.

Moreover, DFMS personnel have taken problems back to USAFA and realized academic benefits, including conference presentations and publications, described in *Academic Benefits* section. Notably, the partnership that we have fostered with HQ AFSPC has been of great benefit to our cadets. The final section describes the advanced studies projects that have resulted from academic sabbaticals.

Description of Sabbaticals and Operational Benefits

DFMS faculty members have been able to leverage their depth of mathematical knowledge to solve difficult operational problems for HQ AFSPC/A9. Moore [4] states that "this visiting professor program has proven to be a valuable collaboration between HQ AFSPC and USAFA. HQ AFSPC has benefited by being able, often for the first time, to solve some of our thorniest analysis challenges for Space acquisition, sustainment and operations."

The subsections below describe the operational problems that DFMS faculty members tackled at HQ AFSPC/A9.

Bradley Warner: 2002-2003

Professor Warner spent a year prior to his sabbatical developing connections, scoping work, and arranging funding to make a sabbatical at HQ AFSPC possible. He accomplished this with the help and vision of HQ AFSPC Col T.S. Kelso, Col Kent Lambert, Col (ret) Greg Keethler, and Lt Col Lee Lehmkuhl. The first purpose of the sabbatical was to understand and gain an appreciation for the work done at an operational Air Force command. The second was to help A9 with their projects. Finally, he sought ways for USAFA and HQ AFSPC to continue to work together after the sabbatical.

Professor Warner spent the first month of his sabbatical learning about the different programs and projects at A9. Likewise, A9 found projects to capitalize on Dr. Warner's expertise. The end result was that he learned about the type of work Air Force analysts perform, contributed on several projects, and developed some opportunities for future collaboration. As summary of the work:

- He assisted in the analysis on a classified project for Task Force Enduring Look that resulted in a novel approach to examining the impact of the Global Positioning System (GPS) errors on the accuracy of Joint Direct Attack Munition (JDAM). The idea was that modeling the distribution of errors for the munition was a hybrid between a discrete distribution and a continuous distribution. If the target was inside a bomb blast crater, then the error was recorded as a zero. If the target was outside the blast crater then a distance could be measured. Modeling this required Dr. Warner's team to estimate parameters for the two different distributions and combine them into a single cumulative distribution function. The weighting of these two distributions also had to be estimated. The result was a predictive model that gave the probability of specified miss distance. This could then be used to determine if assets needed to be used to adjust the GPS satellites.

- He was a team member on the Operationally Responsive Spacelift Analysis of Alternatives. The team developed a novel approach by determining measures of effectiveness and then defining requirements and analyzing possible solutions, alternatives. In developing these measures of effectiveness, the team developed several animated .gif files to help decision makers understand the impact of space assets on operational outcomes. This was an early attempt at visual analytics. The team also assisted in the design of experiments to determine the impact of the different alternatives for measures of effectiveness in campaign level simulations of a futuristic war.

- He assisted the Covariance Accuracy Working Group on developing metrics to summarize satellite tracking error information. Previous work included spherical error regions and elliptical error regions. The spherical error regions were too large especially for low Earth orbit satellites where most of the error was in the direction of the satellite motion. The ellipsoid error regions did not have this problem but also assumed that the errors were independent. The team found a way to quickly find an error ellipsoid using the entire covariance matrix. There was also a question about finding a rectilinear region that incorporated at least as much probability. This was desirable because it was easier to understand and report. This region was determined using a variation of the Bonferroni inequality (ref: `https://en.wikipedia.org/wiki/Boole%27s_inequality`). Figure 1 illustrates the error ellipsoid and corresponding Bon-

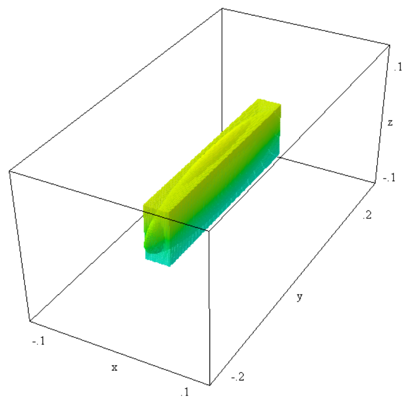

Figure 1: Error ellipsoid and a bonferroni box.

ferroni box for a hypothetical low Earth orbit satellite.

- He taught a number of seminars for the analysts. The topics included visual representation of data, the bootstrap, and categorical data analysis.
- He established links between USAFA and HQ AFSPC that resulted in academic benefits, described in Section 3 below.

As a result of Dr. Warner's work, a memorandum of agreement was established that paved the way for other faculty members to take sabbaticals at HQ AFSPC. For his accomplishments Professor Warner received the U.S. Air Force Award for Exemplary Civilian Service.

Michael Brilleslyper: 2007-2008

Dr. Brilleslyper worked with Mr. Mark Staley of HQ AFSPC/A9 to help develop a database that contains information about the world's terrain. The purpose was to allow more realistic evaluation of GPS performance in different regions. In order for the GPS to function, the user on the ground must have a direct line of sight (LOS) to at least four (of the currently 31) satellites in the GPS constellation. Historically, GPS performance models assumed that a user on the

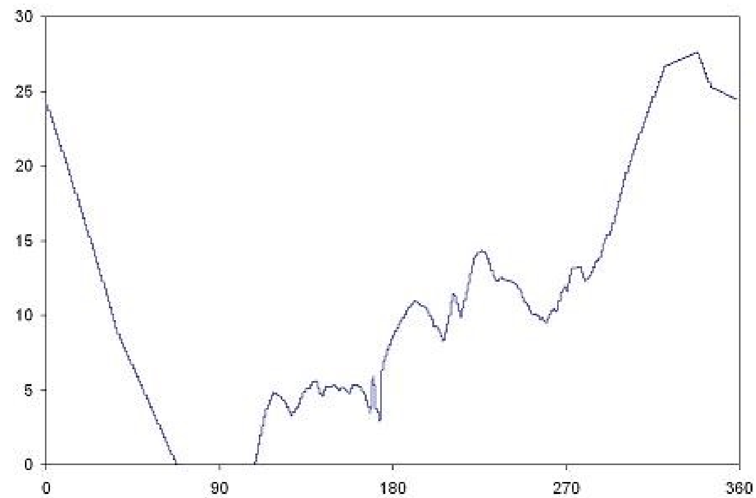

Figure 2: Complete terrain mask for a location in the Colorado Springs foothills: elevation angle in degrees as a function of azimuth.

ground had direct LOS to all points in the sky that were 5 degrees or more above the local horizon. While such an assumption may be appropriate for Kansas, it is certainly not reasonable for the mountains of Afghanistan.

The goal was to build a database that contained complete terrain mask information for every point on Earth that was not "flat." A *terrain* mask is a plot that shows the angle of elevation from a user to every point of that user's local horizon. For example, someone standing at the bottom of a steep canyon might have very large elevation angles in two directions, but much smaller angles looking towards the length of the canyon. Someone standing in the middle of the desert might have a terrain mask where every point on the horizon has an elevation angle of less than 5 degrees. Any location with a terrain maximum elevation angle (MEA) below 5 degrees is considered flat for our purposes. Figure 2 shows a (not flat) terrain mask for a point in the Colorado Springs foothills. (A terrain mask does not display negative elevation angles even though they may occur.)

Dr. Brilleslyper contributed to this effort in the following ways:

- The first significant part of this project was to identify which earth locations are flat. Fortunately, terrain data for most of the world's land masses is freely available; unfortunately, the data is huge. In February 2000, the Space Shuttle Endeavor spent ten days in orbit collecting surface radar returns of the Earth's land masses. Over 12 terabytes of raw data was processed by NASA into roughly 15,000 separate files each of size 2.8 megabytes. This database is referred to as the Shuttle Radar Topography Mission (SRTM) data. This data contains information on about 23 billion *posts* - an earth location with elevation data. Using brute force, to compute the MEA for every post would have been intractable; it would have required *tens of thousands of years* on a fast PC. In contrast, Dr. Brilleslyper and Mr. Staley developed a program that found and stored the MEA for *every location* in the SRTM database in only *a few hours*.

  Their algorithm used a number of techniques for limiting where to look and also for performing calculations very quickly. They employed several geographic ideas that took advantage of the fact that the Earth is curved and tall objects appear to get shorter as one moves further away (eventually all objects disappear over the horizon). This allowed them to limit the search area for each post. They then employed a "divide and conquer" method: they divided the posts into cells of 441 posts each, identified the maximum altitude in each cell, and from that did a small exhaustive search to set up an "interim" maximum elevation angle against which to measure the cells.

  Another key idea in the program for calculating elevation angles was to translate both the user location and the distant post from local North, East, up coordinates into Cartesian Earth-centered, Earth-fixed coordinates (ECEF). In this system, the z-axis runs through the poles, the equatorial plane coincides with the equator and the positive x-axis points through the Prime Meridian. The slope of the elevation angle is found by projecting the difference of the position vectors into the tangent plane of the user to obtain the run and onto the user vector itself to obtain the rise. This allowed them to quickly identify flat cells (MEA less than five degrees) without searching them exhaustively. They incorporated a number of time-saving

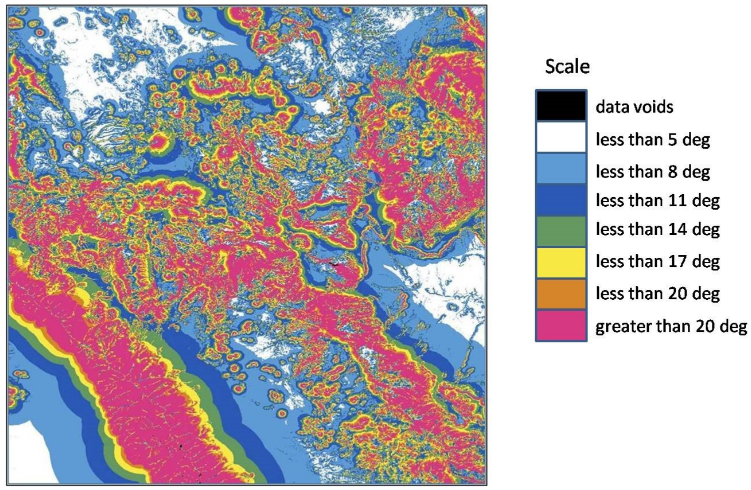

Figure 3: Terrain maximum map: central Rocky Mountains. The Southwest corner is located at 38N, 106W.

techniques, such as comparing rise-over-run (slope) to $\tan(5^\circ)$ rather than using the square root and the arctangent function in their calculations. Using this approach, their program quickly discovered that roughly 17 billion of the 23 billion posts are flat. Hence, *about 74% of the planet's land masses and coastal areas are essentially smooth with regard to LOS applications for satellites.*

- Finding the MEA at every location was only a stepping stone to the ultimate goal of cataloging and storing full terrain masks for locations with potential LOS blockages. However, it yielded an unexpected and fascinating alternative view of the Earth's topography. Figure 3 shows a terrain maximum map of a region in the central part of the Rocky Mountains in Colorado. The different colors correspond to distinct ranges of maximum elevation angles. Note that there is no direct indication of distance or direction to the locations that cause the maximum elevation angle. Figure 4 shows a more standard topographic map of the same region. Note how certain geographic features are reflected in both figures. Each of these two maps represents about 4000 square miles and was generated by over 1.4 million pixels.

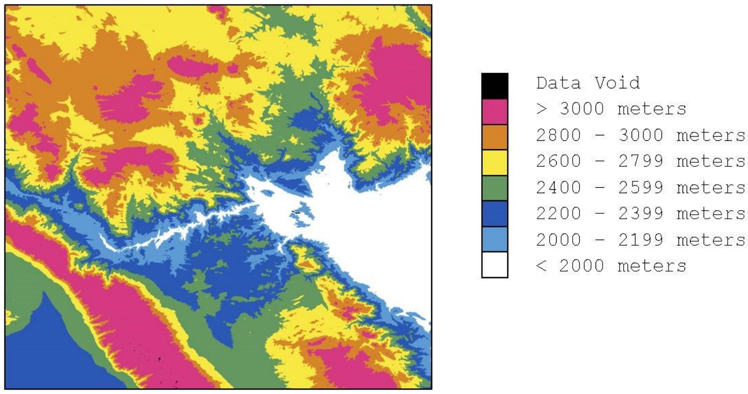

Figure 4: Topographic map: central Rocky Mountains. The Southwest corner is located at 38N, 106W.

Ultimately HQ AFSPC/A9, relying in large part on Dr. Brilleslyper's contributions, realized the entire project's goal, and HQ AFSPC uses the full terrain mask data operationally to provide more robust performance metrics for the GPS in local regions. Moreover, there are additional uses involving constellations of other systems besides the GPS. The project spurred several conference presentations and a technical paper; see the *Academic Benefits* section.

Dr. Brilleslyper's time at HQ AFSPC began a string of three consecutive sabbaticals, all dealing largely with computer modeling of the GPS. This solidified the relationship and continuity between USAFA/DFMS and HQ AFSPC/A9.

Dale Peterson: 2008-2009

Dr. Peterson came into his sabbatical with nearly 10 years prior experience in space systems in the aerospace industry, including satellite control. This made him a good fit to continue with the GPS program to which Professors Warner and Brilleslyper had contributed.

One of the missions of HQ AFSPC/A9 is GPS satellite constellation design. This includes identifying the satellite positioning that leads to the most accurate receiver readings at earth locations. Most

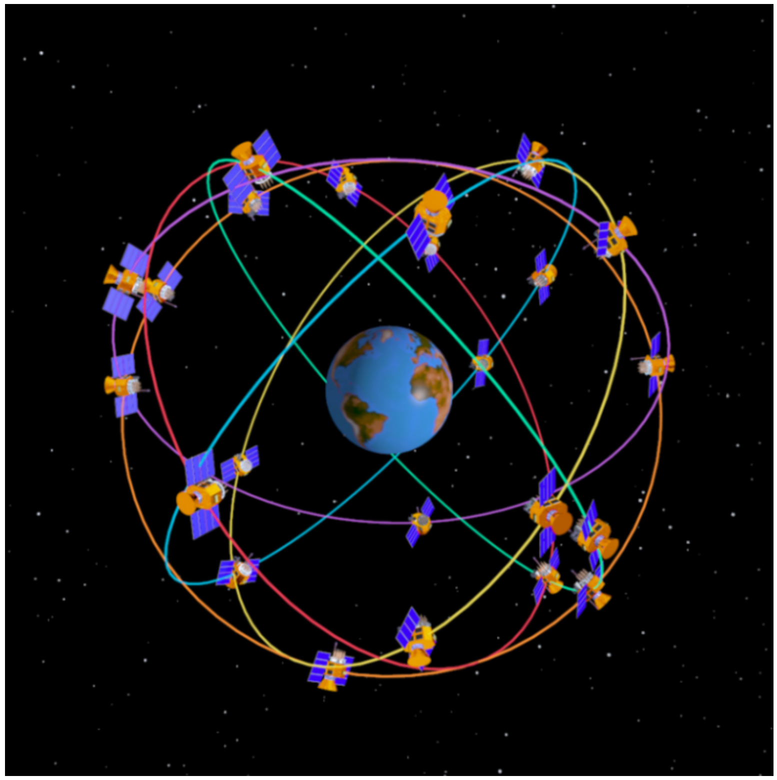

Figure 5: The nominal GPS constellation (figure available at `http://www.gpstk.org/bin/view/Documentation/LinuxJournalPaperSeptember2004`).

of Dr. Peterson's work during this year focused on this mission.

The GPS satellites are placed in six nearly equally-spaced orbital planes, and each plane typically contains 4, 5, or 6 satellites; see Figure 5.

Because of failures and launches, the numbers of satellites in the planes sometime change. Thus, HQ AFSPC/A9 was interested in optimizing performance for all *configurations* - assignments of the number of satellites to each orbital plane. For example, 565544 means there are 5 satellites in the first plane, 6 in the second, and so on. Figure 6 is a *coverage map* for a GPS constellation having this configuration. It displays the *outage* for each geographic location: the time (in bins, e.g. 15-30 minutes) over a day each location's

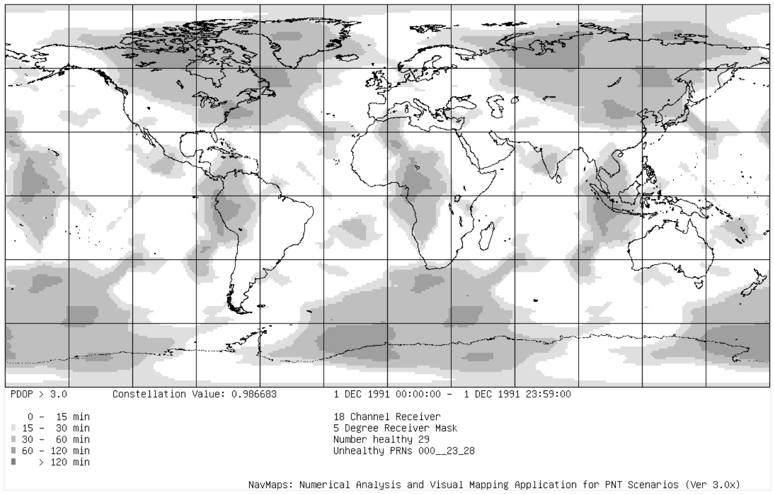

Figure 6: Coverage map for a 565544 configuration.

GPS receiver accuracy reading is below some threshold.

Because one can assign one of three numbers of satellites - 4, 5, or 6 - to each of the six planes, there are 36 or 729 configurations. Optimizing for even one configuration is time consuming, and subsequent performance predictions based on all possible configurations are also time consuming. HQ AFSPC/A9 sought to reduce the number of configurations to be studied.

Mr. Mark Storz of HQ AFSPC/A9 observed that the configuration 655445 - which is 565544 except planes are shifted - has the same overall performance as 565544, just over different locations at different times, when keeping the same satellite placements in corresponding (shifted) planes [11]. The same holds for the "reflection" 445565, when the satellite placements are reflected in corresponding planes. Notice the patterns that look like "turkeys on a serving plate" in Figure 6 are shifted 60 to the west in Figure 7 and Figure 8 is a map of the "mirror image" as indicated by the "drumsticks" pointing to the west instead of the east.

HQ AFSPC/A9 was interested in how many classes of configurations exist. Stated mathematically, this problem is: given the 6-permutations on an alphabet of 3 letters, repeats allowed, how

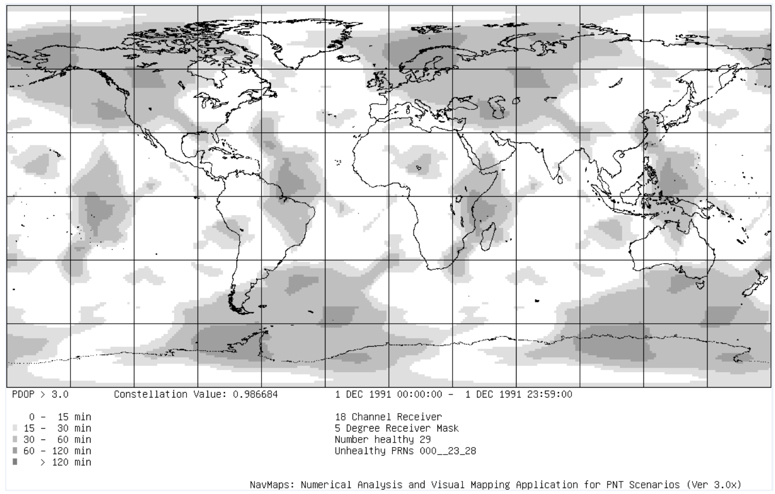

Figure 7: Coverage map for a 655445 configuration (565544 shifted 60 to the west).

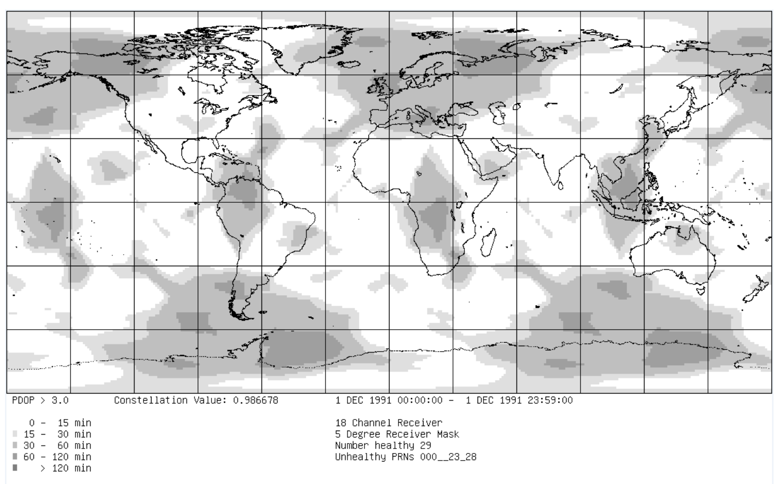

Figure 8: Coverage map for a 445565 configuration (mirror image of 565544).

many classes are there where two permutations are in the same class whenever one can be obtained from the other by a reversal or any number of wrap-around shifts? Dr. Peterson's research background includes combinatorics, so HQ AFSPC/A9 asked him to identify the number of classes. He solved this problem using a combinatorial algebraic result known as Polya's Theorem. He found that there are 92 classes and identified other properties about the classes, such as their sizes and structures [6].

Among the other problems on which Dr. Peterson worked:

- GPS receivers at earth locations may receive up to r channels (satellite signals), but there may be n satellites in view where $r < n$. The problem is to identify which r satellites yield the best performance. A brute force application of the performance equation, which requires matrix multiplications and inversions applied to all $\binom{n}{r}$ combinations at earth locations over time, resulted in numbers of operations that were impractical for computer studies. But the number of operations required was reduced by orders of magnitude by ordering the combinations such that the change between successive combinations consists of a swap of just two satellites, then using a linear algebra technique called the *Sherman-Morrison formula* [7] to update a matrix inverse with only a small number of operations. Such an ordering of combinations is called a *binary-reflected Gray code order* [9]. Dr. Peterson programmed these techniques into an HQ AFSPC/A9 computer model called *Numerical Analysis and Visual Mapping Application* (NAVMAPS). NAVMAPS is used operationally to position satellites.

- A common metric for measuring the effectiveness of a GPS constellation is coverage. This is computed from coverage maps: the outages from all locations are changed from minutes to percentages, subtracted from 100%, and then averaged. HQ AFSPC/A9, however, was interested in a more robust performance function. Dr. Peterson developed a performance function based on a logistic curve that is a continuous function of performance readings, and also weights "trouble" spots. Using this as an objective function in optimization studies reduced the variability in outages [5].

- HQ AFSPC/A9 wished to expand NAVMAPS into a major software tool called *Operational Performance Mapping Availability Program* (OPMAP) that analyzes multiple GPS constellations and displays, based on probabilities of satellite failures and expected launches, expected performance values and maps over time. Dr. Peterson designed the structure and programmed the prototype. He coauthored a Statement of Work (SOW) for a contractor to add interfaces and helped oversee its work. Dr. Boedigheimer continued work on OPMAP; see in the second section of this paper.

For his accomplishments Professor Peterson received the U.S. Air Force Award for Exemplary Civilian Service.

In ongoing collaborative work, Dr. Peterson and one of his former students, Lt Sam Rinaldi, identified an appropriate nonlinear programming algorithm and coded it to find optimal GPS satellite placements.

Ralph Boedigheimer: 2009-2010

Dr. Boedigheimer's specialty is statistics, which became an important need for HQ AFSPC/A9 during this year. Dr. Boedigheimer contributed to several projects.

- He performed a thorough, formal verification of the *Operational Generalized Availability Program* (OpGAP), which is HQ AFSPC's main software application for predicting the satellite functional availability resulting from planned launch schedules and expected satellite functional lifetimes. His deliverables included Technical Report 10-01 [1] and a briefing to the GPS Functional Availability Working Group. The screen shot of one of the OpGAP menus given in Figure 9 is an example of the verification performed. On this screen, several non-functioning windows were noted, input interdependencies were identified, and an ineffective help toggle was detected.

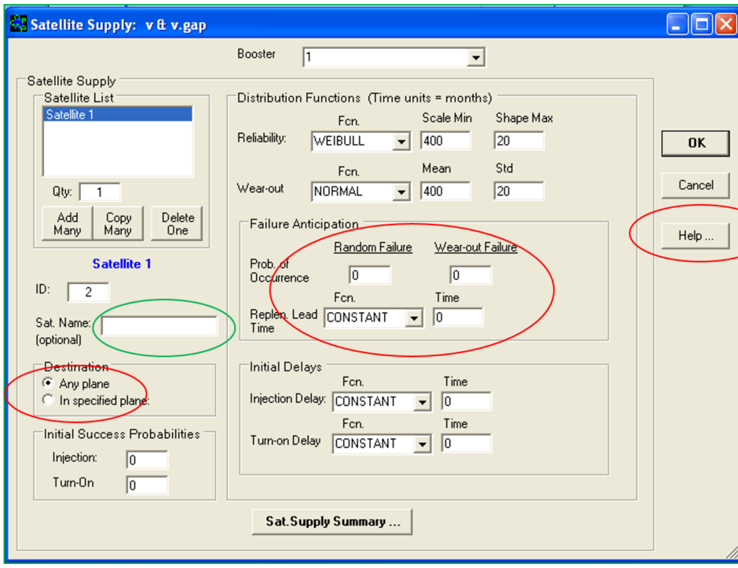

Figure 9: OpGAP menu

- He contributed to the OPMAP software tool begun by Professor Peterson the year before. OPMAP was further developed to translate OpGAP results into maps of warfighter-relevant metrics. For version 1, he completed nearly 200 test cases required to formally accept OPMAP software from the contractor. For the next version, he revised the statistics presented by the software, retooled the software engine needed to compute some of those statistics to incorporate a Monte Carlo approach, and reshaped the way those statistics are displayed for the user. The software provided a more realistic and useful result for the warfighter as can be seen in the example screen in Figure 10. The screen shot shows a world view of GPS satellite coverage (top window), the reliability of each GPS satellite in the constellation (bottom-left window), and a time series of critical constellation statistics as satellites fail and are replaced in a Monte Carlo simulation (bottom right window).

- He studied the sensitivity of OpGAP output to input parameters. Over a 6-month period, he teamed with HQ AFSPC/A9A personnel to completely scrub GPS predictive analysis. Their new analysis and reporting process became the "gold standard" for other satellite programs to follow. The reliability study

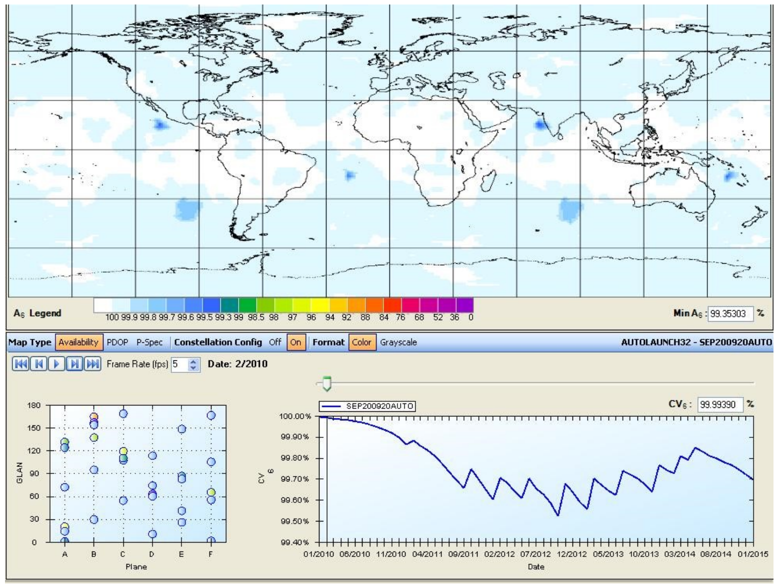

Figure 10: OPMAP screen output.

provided a comprehensive review of the process for developing the four-parameter failure models used to predict individual satellite reliability for the GPS. The final report demonstrated how multiple sources of error affect the variability of the four-parameter failure models used for each satellite and how that variability propagates to the final result. This study removed a pessimistic bias in reliability parameters and provided a solid basis for senior leaders to make informed GPS sustainment and acquisition decisions. Deliverables included Technical Report 11-01 [2] with over 200 pages of computer code in R and a briefing to HQ AFSPC commander General C. Robert Kehler (HQ AFSPC/CC). The key recommendations were:

– Establish the accuracy of the GPS reliability predictions
– Evaluate the constellation using mission reliability
– Generate a verifiable technical failure analysis report biennially
– Establish an approved method to update the random effects model
– Institute a verifiable reporting system for failure data

- Apply rigorous scrutiny to model selection
- Correct specific errors identified by this study
- Apply lessons learned to Block IIF and GPS III satellites

For his accomplishments on the GPS Predictive Analysis Team, Professor Boedigheimer was awarded the Team of the Quarter for HQ AFSPC.

Academic Benefits

Moore [4] states that "the knowledge gained by the visiting professors in performing operational analysis has led to a stronger emphasis of the mathematics curriculum on real-world applications." Benefits to cadets have included:

- In the classroom we have related our *applications of advanced mathematics* to actual Air Force operational missions. These applications have motivated the cadets in their mathematical education in anticipation of their Air Force careers.

- We have expressed the *importance of computer programming* to our mathematics majors. This emphasis increased following Dr. Brilleslyper's sabbatical, which began a string of sabbaticals where advanced mathematical techniques were made operational through computer models.

- We have *brought HQ AFSPC personnel to speak in USAFA courses.* Col Kelso spoke to a general audience, Mr. Bob Morris presented to cadets in Astro 321, and Capt. Mike Warner's advanced astrodynamics course visited NORAD.

- The HQ AFSPC-USAFA collaborations have led to several *cadet advanced study projects*; see the *Description of cadet projects* section.

The academic benefits of these sabbaticals extend beyond pedagogy to scholarly activities. The following conference talks were given by the authors and collaborating HQ AFSPC personnel.

- Warner, Bradley, "Applicability of Equivalence Testing for Comparing OT&E and M&S Data", Military Operations Research Symposium (MORS), Albuquerque, October 2002.
- Boedigheimer, Ralph, Jeffrey Grobman, and Bradley Warner, "Simulation V&V with Bioequivalence Testing", Institute for Operations Research and the Management Sciences (INFORMS) Annual Meeting, Denver, October 24-27, 2004.
- Staley, Mark, and Mike Brilleslyper, "Mapping the Earth with Elevation Angles", Rocky Mountain Section Meeting of the Mathematical Association of America (MAA), Spearfish, SD, April 2008.
- Staley, Mark, and Mike Brilleslyper, "Integrating High Resolution Terrain Data into Global PDOP Calculations", Institute of Navigation, Global Navigation Satellite System (ION GNSS) 2008, Savannah, GA, 19 Sep 2008.
- Staley, Mark, and Mike Brilleslyper, "Mapping the Earth with Elevation Angles", 100th Annual Meeting of the Mathematical Association of America (MAA) - Joint Mathematics Meetings, Special session on Imaging the Earth, San Antonio, TX, 12 Jan 2015.
- Peterson, Dale, "GPS satellite configurations and their astrodynamic and algebraic 'orbits' ", Intermountain/Rocky Mountain joint Regional Conference of the Mathematical Association of America (MAA), Grand Junction, CO, Apr 2016.

Finally, our work resulted in several publications:

- Dr. Warner and Dr. Boedigheimer coauthored a paper on a statistical method applied to validating simulations titled "The Applicability of Equivalence Testing to Simulation Validation" that appeared in a MORS journal [12].
- Dr. Warner coauthored the chapter "Critical Thinking" in the book Methods for Conducting Military Operational Analysis, published by MORS [8].
- Dr. Brilleslyper coauthored a paper on terrain masking titled "Integrating High Resolution Terrain Data into Global PDOP

Calculations" in a prominent journal dedicated to the positioning, navigation, and timing mission that is published by the Institute of Navigation (ION) [10].

- Dr. Peterson coauthored a paper titled "Global Positioning System Performance Optimization using a Normalized Function on Configuration Classes" in a prominent journal published by the The American Institute of Aeronautics and Astronautics (AIAA). The paper presents his performance function to replace coverage and applies it to several configurations [5].
- Dr. Peterson coauthored a paper titled "Group Orbits of GPS Satellite Configurations for Constellation Management" that applied algebraic combinatorial techniques to identify classes of configurations on which to optimize [6].

Descriptions of cadet projects

Beginning 2009-2010 the USAFA Department of Mathematical Sciences (DFMS) instituted a year-long capstone course for senior math majors. The following year it became a major requirement. Some projects are individual research and some are in teams [3]. This course became an excellent vehicle for HQ AFSPC projects.

Below are descriptions of the projects. They were all directed by one of the authors, Dale Peterson, with the support of Mark Storz and Mark Staley of HQ AFSPC/A9. These projects have given cadets valuable operational and mathematical experiences.

Title: Optimizing a Spare GPS Satellite Using Mathematical Programming.
Year: 2009-2010.
Cadet(s): Jacob Belka.
Description: The GPS satellite nominal constellation consists of 24 satellites, four in each of the six equally spaced orbital planes. If we can add a spare satellite, which orbital plane do we pick and where in that plane should we place it for optimal performance? Our approach uses mathematical programming and numerical methods, a technique that does not appear to have been previously applied to GPS. We are comparing our new results with those found by the Air Force

Space Command and contractors using other techniques.
Presentation(s): Southwestern Undergraduate Mathematics Research Conference (SUnMaRC), El Paso, TX, Mar. 2010; Service Academy Student Mathematics Conference (SASMC), USAFA, Apr. 2010.

Title: Universal Time and Leap Seconds in Space Operations.
Year: 2011-2012.
Cadet(s): Jackie Mozingo. Description: The time scale that forms the basis for civil time is known as Coordinated Universal Time (UTC). It slows down in an irregular fashion as the tidal forces change the mass distribution of the earth and slow its rotation rate. Some international stakeholders, e.g. bankers, would like to change to Atomic Time, because it does not vary. But UTC is more convenient for satellite control. Nevertheless, atomic time may be adopted by 2018. Thus it is important to develop tools to evaluate the error that can be expected in operational software that tracks space objects from the ground or ground objects from space. We discuss these tools and how they provide the user a sense of the adverse operational impacts as the biased position deviates more and more from the true position.
Presentation(s): USAFA Department of Mathematical Sciences, Apr. 2012.

Title: Genetic Algorithm for GPS Satellite Positioning.
Year: 2013-2014.
Cadet(s): Zachary Saunders
Description: Develop a genetic nonlinear programming algorithm to optimize the performance of the GPS constellation by improving the positions of the GPS satellites within the constellation.

Title: Chebyshev Missile Trajectory Tool.
Year: 2014-2015.
Cadet(s): Lacey Dreppard, Stuart Evers, and James Kegyes.
Description: The team created a new algorithm for missile trajectory simulation and estimation to serve as a key

contributor to Air Force Space Command. Using Chebyshev polynomials in accordance with the Gauss-Newton algorithm, our team applied an iterative approach to the convergence of a missile trajectory. Applications of this project extend to realms such as scientific research regarding the acquisition of ground-based sensors as well as serving as a tool for national security in predicting harmful possibilities and mitigating threats.
Presentation(s): Service Academy Student Mathematics Conference (SASMC), USAFA, Apr. 2015; Mathematical Association of America (MAA) National MathFest Conference, Washington DC, Aug. 2015.

Title: Missile Trajectory Simulation and Estimation Using Chebyshev Polynomial.
Year: 2015-2016.
Cadet(s): Joshua Bradley, Conner Smith, and Robert Sellers.
Description: This follow-on to the previous year's project improved the trajectory model, including using higher order Chebyshev polynomials, developed an improved seed for the nonlinear program that corrected infeasible trajectories, and allowing the user to input a specific thrust profile. Presentation(s): Service Academy Student Mathematics Conference (SASMC), USMA, Apr. 2016.

Title: Genetic algorithm for GPS satellite positioning.
Year: 2015-2016.
Cadet(s): Felix Knutson, Marika Nemeth, Ron Malloy, and Andrew Vanden Berg.
Description: The Air Force Space Command is interested in finding an optimal GPS satellite constellation to provide the best performance. To determine this optimal constellation, the team implemented a genetic algorithm that evaluates the precision of a constellation by analyzing the location of points around the globe over the satellite's period and iteratively selects constellations that are most "fit" by employing mutation and crossover.
Presentation(s): Service Academy Student Mathematics

Conference (SASMC), USMA, Apr. 2016; Mathematical Association of America (MAA) Regional Conference, Colorado Mesa University, Grand Junction, CO, Apr. 2016.

Title: Modeling missile trajectories and radar suites.
Year: 2016-2017.
Cadet(s): Lauren Bramblett, Cinthya Elizondo Gamez, and Juan Orozco.
Description: This is follow-on to the past two years' projects. Verify missile trajectory model, adjust missile profiles to account for changing thrust, and incorporate radar observation errors.
Presentation(s): Anticipated: Service Academy Student Mathematics Conference (SASMC), USNA, Apr. 2017.

Additionally, in 2012-2013, Mark Storz and Dr. Peterson worked with casual status 2nd Lieutenant Samuel Rinaldi to implement a simulation and optimization algorithm to optimally position GPS satellites. Lt. Rinaldi identified an appropriate nonlinear programming method, incorporated it into a GPS computer simulation, and performed extensive analysis on the outputs. Lt. Rinaldi presented their work to the Air Force Operational Test & Evaluation Center Detachment 4 (AFOTEC DET4 - space and missiles component), Peterson Air Force Base, Colorado Springs, CO. This work culminated in a publication [5].

Conclusion

Beginning with Dr. Warner, USAFA/DFMS and HQ AFSPC/A9 have established a strong connection that has paid off in both operational benefits to HQ AFSPC and academic benefits to USAFA. Some of the operational benefits include examining the effects of GPS errors on JDAM, increased understanding of the impact of space assets on operations, and the use of graphical tools such as error ellipsoids applied to satellite tracking (Dr. Warner); a comprehensive terrain mask map of the Earth that benefits in particular the assessment of GPS performance (Dr. Brilleslyper); solutions of problems in GPS satellite constellation design and the development of OPMAP, a comprehensive GPS performance prediction tool (Dr. Peterson); and upgrades to the OpGAP tool for predicting the satellite functional availability and sophisticated statistical techniques applied to the OPMAP tool (Dr. Boedigheimer).

Academic benefits have included six conference presentations, five publications, and several advanced study projects for our mathematics majors. These projects have given our cadets a significant experience with an operational Air Force command. The types of problems on which they have worked have included GPS constellation design, leap second effects on space operations, and missile defense / radar study.

We expect this trend to continue with more cadet projects, continuing collaboration, and possibly more sabbaticals.

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